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Fuzzy Dot Satisfactory Bck- Filters

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Abstract: The concept of fuzzy dot subalgebra of *BCK*/*BCI*-algebras was introduced by Jun and Hong paper [8]. In 2003 Jun [7] was introduced the concept of satisfactory *BCK*-filters. The concept of fuzzy satisfactory *BCK*-filters was introduced by Najati [13]. In this paper, we define the notion of fuzzy dot satisfactory *BCK*-filters and investigate some of its properties.

Keywords: BCK-algebras, filter, dot filter, satisfactory filter, fuzzy filter, fuzzy dot filter, fuzzy satisfactory filter, fuzzy dot satisfactory filter.

1. Introduction

The notion of BCK-algebra was introduced by Imai and Iseki in 1966 [5]. In the same year, Iseki [6] introduced the notion of BCI-algebra which is a generalization of a BCK-algebra. After the introduction of the concept of fuzzy sets by Zadeh [17], several researchers were conducted on the generalization of the notion of fuzzy sets. Jun and Hong in [8] introduced the notion of a fuzzy dot subalgebra of a BCK/BCI-algebras as a generalization of a fuzzy subalgebra and state a condition for a fuzzy dot subalgebra and prove several basic properties which are related to fuzzy dot subalgebra. In [12] Najati introduced the notion of fuzzy dot filters in BCK-algebras and investigate some of its properties. In [13] Najati introduced the notion of a fuzzy satisfactory filter in BCK-algebra and prove several basic properties which are related to fuzzy satisfactory filter. In this paper, we introduce the notion of fuzzy dot satisfactory BCK-filter and prove some their fundamental properties, and then we state a condition for a fuzzy subset to be a fuzzy dot satisfactory filter.

2. Preliminaries

We review some basic definitions and properties that will be useful in our results. A *BCK-algebra* X is defined to be an algebra $(X,*,0)$ of type $(2,0)$ satisfying the following conditions.

$BCK-1$ $\big((xy)(xz)\big)(zy)=0$,

$BCK-2$ $\big(x(xy)\big)y=0$,

$BCK-3$ $xx=0$,

$BCK-4$ $0x=0$,

$BCK-5$ $xy=0$ and $yx=0 \Rightarrow x=y$,

for all $x,y,z\in X$, where $xy=x*y$, and $xy=0$ if and only if $x\le y$.

In a *BCK*-algebra X, the following properties hold for all $x,y,z\in X$:

P-1 $x0=x$,

P-2 $(xy)z=(xz)y$,

P-3 $x\le y$ implies that $xz\le yz$ and $zy\le zx$,

P-4 $(xz)(yz)\le xy$,

P-5 $x\le y, y\le z\Rightarrow x\le z$,

P-6 $x\big(x(xy)\big)=xy$,

P-7 $xy\le x$.

A *BCK*-algebra X satisfying the identity $x(xy)=y(yx)$, for all $x,y\in X$, is said to be *commutative*. If there is a special element 1in a *BCK*-algebra X satisfying $x\le 1$, for all $x\in X$, then $1$is called a *unit* of X. A *BCK*-algebra X with unit is said to be *bounded*. In what follows let X denote a bounded *BCK*-algebra unless otherwise specified, and we will use the notation x^* instead $1x$ for all $x\in X$. In a bounded *BCK*-algebra X we have:

P-8 $1^*=0$ and $0^*=1$.

P-9 $y\le x$ implies that $x^*\le y^*$.

P-10 $x^*y^* \leq yx$.

If X is a commutative bounded *BCK*-algebra, then the equalities $(x^*)^* = x$, $x^*y^* = yx$ hold, for all $x, y \in X$.

We review some fuzzy concepts. A fuzzy subset of a nonempty set X is a function $\mu: X \to [0,1]$. We shall use the notation X_μ for $\{x \in X | \mu(x) = \mu(1)\}$. The set $\mu_t = \{x \in X | \mu(x) \geq t\}$, where $t \in [0,1]$, is called the *t-level subset* of μ .

A nonempty subset of X Then F is called a *filter* of X, if it satisfies the conditions.
F-1 $1 \in X$,
F-2 $(x^*)^*y^* \in F, y \in F \Rightarrow x \in F$ for all $x, y \in X$.

A nonempty subset F of X is called a *satisfactory filter* of X, if it satisfies (F-1) and
F-3 $(x(y(yz)^*)^*)^* \in F, x \in F \Rightarrow (yz)^* \in F$, for all $x, y, z \in X$.

A *BCK*-algebra X satisfying $(xz)(yz) = (xy)z$, for all $x, y, z \in X$, is said to be *positive implicative*. A *BCK*-algebra X is positive implicative if and only if it satisfies $xy = (xy)y$, for all $x, y \in X$.

A fuzzy subset $\mathcal{F}$ in X is said to be a *fuzzy filter* of X, if it satisfies:
FF-1 $\mathcal{F}(1) \geq \mathcal{F}(x)$,
FF-2 $\mathcal{F}(x) \geq min\{\mathcal{F}((x^*y^*)^*), \mathcal{F}(y)\}$, for all $x, y \in X$.
Note that every fuzzy filter is order preserving (see [9, Proposition 3.5]).

Definition 2.1. [12] A fuzzy subset $\mathcal{F}$ in X is said to be a *fuzzy dot filter* of X, if it satisfies FF-1 and,
FF-3 $\mathcal{F}(x) \geq \mathcal{F}((x^*y^*)^*)\mathcal{F}(y)$, for all $x, y \in X$.

Theorem 2.2. [12, Theorem 3.7] *Let* $\mathcal{F}$ *be* a fuzzy subset of X and $\mathcal{F}(1) = 1$. Then

(i) $\boldsymbol{\mathcal{F}(xz)^* \geq \mathcal{F}(y)}$ implies $\boldsymbol{\mathcal{F}(z) \geq \mathcal{F}(y)}$,

(ii) $\boldsymbol{xy \leq z}$ implies $\boldsymbol{\mathcal{F}(y) \geq \mathcal{F}(x)\mathcal{F}(z^*)}$.

Theorem 2.3. Let X be commutative and let $\mathcal{F}$ be a fuzzy dot filter of X . Then for all $x, y \in X$

$$\mathcal{F}(x) \geq \mathcal{F}((xy)^*x)^*)$$

Proof. Let $\mathcal{F}$ be a fuzzy dot filter of X . Then for all $x, y, z \in X$, we get:

$$\begin{aligned} ((xy)^*x)^*(yx)^* &= (yx)((xy)^*x) \\ &\leq y(xy)^* \\ &= (xy)y^* \\ &= (xy)(1y) \\ &\leq x1 \\ &= 0 \end{aligned}$$

Then

$$\mathcal{F}((xy)^*x)^* \leq \mathcal{F}(yx)^*$$

Then by Theorem 2.2 we get:

$$\mathcal{F}(x) \geq \mathcal{F}((xy)^*x)^*)$$

Definition 2.4. [13] A fuzzy subset $\mathcal{F}$ in X is said to be a *fuzzy satisfactory filter* of X, if it satisfies FF-1, and
FF-4 $\mathcal{F}((yz)^*) \geq min\{\mathcal{F}((x(y(yz)^*)^*)^*), \mathcal{F}(x)\}$, for all $x, y, z \in X$.

3. FUZZY DOT SATISFACTORY FILTERS

Definition 3.1. A fuzzy subset $\mathcal{F}$ in X is said to be a *fuzzy dot satisfactory filter* of X, if it satisfies FF-1, and
FF-5 $\mathcal{F}((yz)^*) \geq \mathcal{F}((x(y(yz)^*)^*)^*)\mathcal{F}(x)$, for all $x, y, z \in X$.

Example 3.2. Let $X = \{0, a, b, 1\}$ be a bounded *BCK*-algebra with $*$ defined by

$*$	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

Define $\mathcal{F}$ of X by $\mathcal{F}(1) = 0.4$ and $\mathcal{F}(0) = \mathcal{F}(a) = \mathcal{F}(b) = 0.3$. Routine calculations give that $\mathcal{F}$ is not a fuzzy filter of X. Also, a fuzzy subset υ of X defined by $\upsilon(0) = 0.3, \upsilon(1) = \upsilon(b) = 0.5$, and $\upsilon(a) = 0.6$. Routine calculations give that υ is not a fuzzy filter of X.

Remark 3.3. Every fuzzy satisfactory filter of X is a fuzzy dot satisfactory filter of X , since:

$$\begin{aligned} \mathcal{F}((xz)^*) &\geq \min\{\mathcal{F}((x(y(yz)^*)^*)^*), \mathcal{F}(x)\} \\ &\geq \mathcal{F}_F((x(y(yz)^*)^*)^*)\mathcal{F}(x) \end{aligned}$$

But the converse may not be true as seen in above example, since υ is a fuzzy dot satisfactory filter of X, but it is not a fuzzy satisfactory filter because:

$$\begin{aligned} \upsilon(10)^* &= 0.3 \\ &< \min\{\upsilon((b(1(10)^*)^*)^*), \upsilon(b)\} \\ &= \min\{\upsilon(a), \upsilon(b)\} = 0.5 \end{aligned}$$

Remark 3.4. If F be a nonempty satisfactory filter of X, and the fuzzy set $\mathcal{F}_F$ of X, which defined by $\mathcal{F}_F(x) = s$ if $x \in F$ and $\mathcal{F}_F(x) = t$ if $x \notin F$, for all $s, t \in [0, 1]$ with $s > t$, then by [13, Theorem 4.4] $\mathcal{F}_F$ is a fuzzy satisfactory filter of X. So, by above remark $\mathcal{F}_F$ is a fuzzy dot satisfactory filter of X. In the following theorem we refer this remark with another proof.

Theorem 3.5. Let F be a nonempty satisfactory filter of X and let the fuzzy set $\mathcal{F}_F$ of X, which defined by $\mathcal{F}_F(x) = s$ if $x \in F$ and $\mathcal{F}_F(x) = t$ if $x \notin F$, for all $s, t \in [0,1]$ with $s > t$, then it is a fuzzy dot satisfactory filter of X.

Proof. Let F be a nonempty satisfactory filter of X, since $1 \in F$, we have $\mathcal{F}(1) = s \geq \mathcal{F}(x)$ for all $x \in X$. Now let $x, y, z \in X$. If $(x(y(yz)^*)^*)^* \in F$ and $x \in F$, then $(yz)^* \in F$, then

$$\mathcal{F}_F(yz)^* = s \geq \mathcal{F}_F((x(y(yz)^*)^*)^*)\mathcal{F}_F(x)$$

If $(x(y(yz)^*)^*)^* \notin F$ or $x \notin F$, then

$$\mathcal{F}_F((x(y(yz)^*)^*)^*)\mathcal{F}_F(x) = st \leq \mathcal{F}_F(yz)^*$$

It follows that $\mathcal{F}_F$ is a fuzzy dot satisfactory filter of X.

Theorem 3.6. Let $\mathcal{F}$ be a fuzzy dot satisfactory filter of X, then $X_{\mathcal{F}} = \{x \in X | \mathcal{F}(x) = 1\}$ is either empty or satisfactory filter of X.

Proof. Suppose that $\mathcal{F}$ is a fuzzy dot satisfactory filter of X, clearly $1 \in X_{\mathcal{F}}$. Now let $X_{\mathcal{F}} \neq \emptyset$. Let $x, y, z \in X$, and let $(x(y(yz)^*)^*)^* \in X_{\mathcal{F}}$ and $x \in X_{\mathcal{F}}$, then:

$$\mathcal{F}(yz)^* \geq \mathcal{F}((x(y(yz)^*)^*)^*)\mathcal{F}(x) = 1$$

So $(yz)^* \in X_{\mathcal{F}}$, then $X_{\mathcal{F}}$ is a satisfactory filter of X.

Proposition 3.7. Let $\mathcal{F}$ be a fuzzy dot satisfactory filter of X. Then for all $x \in X$, $\mathcal{F}(0) \geq \mathcal{F}(x^*)\mathcal{F}(x)$.

Proof. Let $x \in X$, then:

$$\mathcal{F}(0) = \mathcal{F}(10)^* \geq \mathcal{F}(x(1(10)^*)^*\mathcal{F}(x) = \mathcal{F}(x^*)\mathcal{F}(x).$$

Lemma 3.8. [7, Theorem 3.13] Let X be commutative and $G^* = \{x^* | x \in G\}$ for every nonempty subset G of X. Then G^* is a satisfactory filter of X if and only if G is a positive implicative ideal of X.

Theorem 3.9. Assume that X is commutative. Let G be a nonempty subset of X and $\mathcal{F}_{G^*}$ a fuzzy set in X defined by:

$$\mathcal{F}_{G^*} = \begin{cases} s & if \ x \in G^* = \{x^* | x \in G\}, \\ t & otherwise, \end{cases}$$

for all $x \in X$ and $0 \leq t < s \leq 1$. If G is a positive implicative ideal of X, then $\mathcal{F}_{G^*}$ is a fuzzy dot satisfactory filter of X.

Proof. Assume that G is a positive implicative ideal of X, then by Lemma 3.8. G^* is a satisfactory filter of X, So $\mathcal{F}_{G^*}$ is a fuzzy dot satisfactory filter of X by Theorem 3.5.

Theorem 3.10. Let $\mathcal{F}$ be a fuzzy set of X. If $\mathcal{F}_t$ is a satisfactory filter of X for all $t \in [0,1]$ where $\mathcal{F}_t \neq \emptyset$, then $\mathcal{F}$ is a fuzzy dot satisfactory filter of X.

Proof. Assume that $\mathcal{F}_t$ is a satisfactory filter of X for all $t \in [0,1]$, and $\mathcal{F}_t \neq \emptyset$, we claim that (FF-1) and (FF-4) are true. If (FF-1) is not true, then there is $x_0 \in X$ be such that, $\mathcal{F}(1) < \mathcal{F}(x_0)$. Put $t_0 = \frac{1}{2}\{\mathcal{F}(1) + \mathcal{F}(x_0)\}$. Then $t_0 \in [0,1]$ and $\mathcal{F}(1) < t_0 < \mathcal{F}(x_0)$, which implies that $x_0 \in \mathcal{F}_{t_0}$ and $\mathcal{F}_{t_0} \neq \emptyset$. So $\mathcal{F}_{t_0}$ is a satisfactory filter of X by assumption. It follows from $1 \in \mathcal{F}_{t_0}$ that $\mathcal{F}(1) \geq t_0$ which is a contradiction. Therefore (FF-1) holds. Suppose that (FF-4) is false. Then there are $x_0, y_0, z_0 \in X$ such that:

$$\mathcal{F}((y_0z_0)^*) < \mathcal{F}((x_0(y_0(y_0z_0)^*)^*)^*)\mathcal{F}(x_0)$$

Taking

$s_0 = \frac{1}{2}\{\mathcal{F}((y_0z_0)^*) + \mathcal{F}((x_0(y_0(y_0z_0)^*)^*)^*)\,\mathcal{F}(x_0)\}$,

we get $s_0 \in [0,1]$ and

$$\mathcal{F}((y_0z_0)^*) < s_0 < \mathcal{F}((x_0(y_0(y_0z_0)^*)^*)^*)\,\mathcal{F}(x_0)$$

It follows from the right-hand side of the above inequality that:

$$(x_0(y_0(y_0z_0)^*)^*)^* \in \mathcal{F}_{s_0} \ and \ x_0 \in \mathcal{F}_{s_0}$$

Since $\mathcal{F}_{s_0}$ is a satisfactory filter of X it follows form $(y_0z_0)^* \in \mathcal{F}_{s_0}$, that $\mathcal{F}((y_0z_0)^*) \geq s_0$ so, which is impossible. Hence (FF-4) is also valid. Consequently, $\mathcal{F}$ is a fuzzy dot satisfactory filter of X.

Theorem 3.11. [12, Theorem 3.8] Let X be a commutative and $\mathcal{F}$ and a fuzzy subset of X. Then $\mathcal{F}$ fuzzy dot filter of X if and only if it satisfies for all $x, y \in X$, $\mathcal{F}(x) \geq \mathcal{F}(yx)^*\,\mathcal{F}(y)$.

Theorem 3.12. If X is commutative, then every fuzzy dot satisfactory filter of X is a fuzzy dot filter of X

Proof. Let $\mathcal{F}$ be a fuzzy dot satisfactory filter of X and $x, y \in X$. Since $x^{**} = x$, we get $(xy)^* = (x(y^{**})^{**})^*$. it follows from FF-5 that:

$$\begin{aligned} \mathcal{F}(y) &= \mathcal{F}(y^{**}) \\ &\geq \mathcal{F}(x(1(1y)^*)^*)^*\,\mathcal{F}(x) \\ &= \mathcal{F}(xy)^*\,\mathcal{F}(x) \end{aligned}$$

So, from Theorem 3.10. that $\mathcal{F}$ is a fuzzy dot filter of X.

The converse of above theorem may not be true as seen in the following example.

Example 3.13. Let $X = \{0, a, b, 1\}$ be a bounded *BCK*-algebra with $*$ defined by:

*	0	a	b	1
0	0	0	0	0
a	a	0	0	0
b	b	a	0	0
1	1	b	a	0

Define $\mathcal{F}$ of X by $\mathcal{F}(1) = 1$ and $\mathcal{F}(x) = t$, $t \in [0,1]$ for all $x \neq 1$. Routine calculations give that $\mathcal{F}$ is a fuzzy dot filter of X, but it is not fuzzy dot satisfactory filter of X, because.

$$\mathcal{F}(ba)^* = t \geq 1 = \mathcal{F}((1(b(ba)^*)^*)^*)\mathcal{F}(1)$$

Theorem 3.14. [12, Proposition 3.4] Every fuzzy dot filter $\mathcal{F}$ of X with $\mathcal{F}(1) = 1$ is order preserving.

Remark 3.15. By Theorem 3.12 and Theorem 3.14 if X is commutative, and $\mathcal{F}$ is a fuzzy dot satisfactory filter of X with $\mathcal{F}(1) = 1$, then it is a fuzzy dot filter, so it is preserving.

We give conditions for a fuzzy dot filter to be a fuzzy dot satisfactory filter of X .

Theorem 3.16. Let X be a commutative and positive implicative. Then every fuzzy dot filter of X is a fuzzy dot satisfactory filter of X .

Proof. Let $\mathcal{F}$ be a fuzzy dot filter of X . Since

$(yz)^* = (z^*y^*)^* = ((z^*y^*)y^*)^* = ((yz)y^*)^* = (y(yz)^*)^*$ for all $y, z \in X$, it follows from Theorem 3.10.

$$\mathcal{F}\ ((yz)^*) = \mathcal{F}((y(yz)^*)^*) \geq \mathcal{F}((x(y(yz)^*)^*)^*)\mathcal{F}(x)$$

So that $\mathcal{F}$ is a fuzzy dot satisfactory filter of X .

Theorem 3.17. Let X be a commutative and let $\mathcal{F}$ be a fuzzy dot satisfactory filter of X where $\mathcal{F}(1) = 1$. Then for all $x, y \in X$

$$\mathcal{F}\ ((xy)^*) \geq \mathcal{F}((x(xy)^*)^*) \qquad (1)$$

Proof. Let $\mathcal{F}$ be a fuzzy dot satisfactory filter of X , then

$$\begin{aligned}\mathcal{F}\ ((xy)^*) &\geq \mathcal{F}((1(x(xy)^*)^*)^*)\mathcal{F}(1)\\ &= \mathcal{F}((((x(xy)^*)^*)^*)^*)\\ &= \mathcal{F}((x(xy)^*)^*)\end{aligned}$$

for all $x, y \in X$. Therefore (1) holds.

Theorem 3.18. Every fuzzy dot filter of X satisfying (1) is a fuzzy dot satisfactory filter of X .

Proof. Let $\mathcal{F}$ be a fuzzy dot filter of X satisfying (1), then by Theorem 3.11.

$$\mathcal{F}\ ((yz)^*) = \mathcal{F}((y(yz)^*)^*) \geq \mathcal{F}((x(y(yz)^*)^*)^*)\ \mathcal{F}(x)$$

for all $x, y, z \in X$. Therefore, $\mathcal{F}$ is a fuzzy dot satisfactory filter of X .

Theorem 3.19. Let X be a commutative and let $\mathcal{F}$ be a fuzzy dot filter of X . Then for all $x, y \in X$

$$\mathcal{F}\ (x) \geq \mathcal{F}((xy)^*x)^*)$$

Proof. Let $\mathcal{F}$ be a fuzzy dot filter of X , then:

$$\begin{aligned}((xy)^*x)^* &= (yx)((xy)^*x)\\ &\leq y(xy)^*\\ &= (xy)y^*\\ &= (xy)(1y)\\ &\leq x1\\ &= 0\end{aligned}$$

for all $x, y \in X$, then $\mathcal{F}\ ((xy)^*x)^* \leq \mathcal{F}\ (yx)^*$, then by Theorem 2.2 (i) we get:

$$\mathcal{F}\ (x) \geq \mathcal{F}((xy)^*x)^*)$$

Theorem 3.20. Let X be a commutative and let $\mathcal{F}$ be a fuzzy dot filter of X where $\mathcal{F}(1) = 1$. Then the following are equivalent for all $x, y, z \in X$

i. $\mathcal{F}$ is a fuzzy dot satisfactory filter of X ,

ii. $\mathcal{F}\ (y) \geq \mathcal{F}((x(yz)^*y))^*)\mathcal{F}(x)$.

Proof. $(i) \Rightarrow (ii)$ Let $\mathcal{F}$ be a fuzzy dot satisfactory filter of X , by Theorem 3.11 and Theorem 3.19 we have

$$\mathcal{F}\ (y) \geq \mathcal{F}((yz)^*y)^*)\ \geq \mathcal{F}(x((yz)^*y)^*)\mathcal{F}(x)$$

Thus (ii) is valid.

$(ii) \Rightarrow (i)$ Assume that a fuzzy dot filter $\mathcal{F}$ of X satisfies the condition (ii), for all $x, y \in X$, it follows from (ii) that:

$$\mathcal{F}\ (x) \geq \mathcal{F}((((xy)^*x)^*)^{**})\mathcal{F}(1)\ \geq \mathcal{F}(((xy)^*x)^*)$$

Thus

$$\mathcal{F}\ (x) \geq \mathcal{F}(((xy)^*x)^*) \qquad (1^*)$$

for all $x, y \in X$. Taking $x = (xy)^*$ in (1^*) implies that

$$\mathcal{F}\ (xy)^* \geq \mathcal{F}(((xy)^*y)^*(xy)^*)^* \qquad (2^*)$$

On the other hand, note that:

$$\begin{aligned}(((xy)^*y)^*(xy)^*)^* &= ((xy)((xy)^*y))^*\\ &= ((y^*x^*)(y^*(xy)))^*\\ &= ((y^*\big(y^*(xy)\big))x^*)^*\\ &= (((xy)((xy)y^*))x^*)^*\\ &= (((y^*x^*)((y^*x^*)y^*))x^*)^*\\ &= (((y^*x^*)x^*)((y^*x^*)y^*))^*\\ &= (((y^*x^*)x^*)((y^*y^*)x^*))^*\\ &= (((y^*x^*)x^*)((0)x^*))^*\\ &= (((y^*x^*)x^*)0)^*\\ &= (((y^*x^*)x^*))^*\\ &= (x(xy)^*)^*\end{aligned}$$

It follows from (2^*) that $\mathcal{F}(xy)^* \geq \mathcal{F}(x(xy)^*)^*$, so from Theorem 3.18 that $\mathcal{F}$ is a fuzzy dot satisfactory filter of X .

Theorem 3.21. Let $\{\mathcal{F}_i\}$, where $i \in N$ be a family of fuzzy dot satisfactory filter of X , then so is $\cap_{i\in I} \mathcal{F}_i$.

Proof. Let for all $x, y \in X$, we get

$$\begin{aligned}\cap_{i\in I} \mathcal{F}_i(1) &= min_{i\in I}\{\mathcal{F}_i(1)\}\\ &\geq\ min_{i\in I}\{\mathcal{F}_i(x)\}\\ &= \cap_{i\in I} \mathcal{F}_i(x)\end{aligned}$$

$$\begin{aligned}\cap_{i\in I} \mathcal{F}_i(yz)^* &= min_{i\in I}\{\mathcal{F}_i(yz)^*\}\\ &\geq\ min_{i\in I}\{\mathcal{F}_i(x(y(yz)^*)^*)^*\ \mathcal{F}_i(x)\}\\ &\geq (min_{i\in I}\{\mathcal{F}_i(x(y(yz)^*)^*)^*\})(min_{i\in I}\{\mathcal{F}_i(x)\}\\ &= (\cap_{i\in I} \mathcal{F}_i(x(y(yz)^*)^*)^*)(\cap_{i\in I} \mathcal{F}_i(x))\end{aligned}$$

Hence $\cap_{i\in I} \mathcal{F}_i$ is a fuzzy dot satisfactory filter of X .

In the following example we can see if $\{\mathcal{F}_i\}$, where $i \in N$ is a family of fuzzy dot satisfactory filter of X, then $\cup_{i\in I} \mathcal{F}_i$ may not be a fuzzy dot satisfactory filter of X.

Example 3.23. Let $X = \{0, a, b, 1\}$ be a bounded *BCK*-algebra with $*$ defined as in Example 3.2. and let fuzzy dot satisfactory filter ν of X which defined by $\nu(0) = 0.3$, $\nu(a) = 0.4$ and $\nu(b) = \nu(1) = 0.5$. Defined the fuzzy subset $\lambda\ of\ X$ by $\lambda(1) = \lambda(a) = 1$ and $\lambda(0) = \lambda(b) =$

0.1 . Routine calculations give that λ is a fuzzy dot filter of X. But $\nu \cup \lambda$ is not fuzzy dot filter, because:
$(\nu \cup \lambda)(10)^* = \max\{\nu(10)^*, \lambda(10)^*\} = 0.3$
$< \nu \cup \lambda((a(1(10)^*)^*)^*)\nu \cup \lambda(a)$
$= \max\{\nu((a(1(10)^*)^*)^*), \lambda((a(1(10)^*)^*)^*)\} \max\{\nu(a), \lambda(a)\}$
$= (\max\{0.1, 0.5\})(\max\{1,0.4\}\, 0) = (0,5)(1) = 0.5$

4. Conclusion

The concept of fuzzy dot subalgebra of *BCK*/*BCI*-algebras was introduced by Jun and Hong paper [8]. In 2003 Jun [7] was introduced the concept of satisfactory *BCK*-filters. The concept of fuzzy satisfactory *BCK*-filters was introduced by Najati [13]. In this paper, we define the notion of fuzzy dot satisfactory *BCK*-filters and investigate some of its properties.

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